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TOWARDS A PRECISE CHARACTERIZATION OF THE COMPLEXITY OF UNIVERSAL AND NONUNIVERSAL TURING MACHINES*

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Abstract. A computation universal Turing machine, U, with 2 states, 4 letters, 1 head and 1 two-dimensional tape is constructed by a translation of a universal register-machine language into networks over some simple abstract automata and, finally, of such networks into U. As there exists no universal Turing machine with 2 states, 2 letters, 1 head and 1 two-dimensional tape only the 2-state, 3-letter case for such machines remains an open problem. An immediate consequence of the construction of U is the existence of a universal 2-state, 2-letter, 2-head, 1 two-dimensional tape Turing machine, giving a first sharp boundary of the necessary complexity of universal Turing machines.

Key words. Turing-machines, two-dimensional tape, universal, nonuniversal, complexity-measures, register-machines, networks of abstract automata

Introduction. About fifteen years ago several attempts were made to find simple universal machines. Well known are the results of Watanabe (1961)—there exists a universal Turing machine with 8 states and 5 letters—and Minsky (1962)—there exists a universal Turing machine with 7 states and 4 letters. However, the gap in complexity between universal Turing machines and Turing machines which are not candidates for universal computation (for example, 2 states in the case of quadruple instructions cannot be sufficient, Fischer (1965)) was quite large and could not be closed. Some generalizations of Turing machines, such as Turing machines with multi-dimensional tape (or tapes) and multiple heads, led to some progress. Hooper (1963) showed the existence of universal Turing machines with 1 state, 2 letters, but also 4 heads, while Wagner (1973) proved Turing machines with 8 states and 4 letters operating on a two-dimensional tape, to be universal. The last result was improved by Kleine Büning and Ottmann (1977) to a universal two-dimensional Turing machine with 3 states and 6 letters and Kleine Büning (1977) proved in his Ph.D. thesis that 2 states and 5 letters or 10 states and 2 letters are also sufficient.

Kleine Büning and Ottmann proved their results by simulating so-called normed networks. This technique was first used by the author (Priese (1974)) to find some simple undecidable Thue-systems. We follow the principles of this technique in this paper also. Wagner's technique is very closely related to the common methods in 1-dimensional Turing machines.

Table 1 gives a review of the existing results. The mentioned complexity is explained in the final section of this paper.

Wagner (1973) proved also that for no dimension n can a 2-state, 2-letter, n-dimensional Turing machine be universal. This last result may offer a surprising chance to close the gap of complexity between universal and nonuniversal Turing machines: In this paper we prove the existence of a universal two-dimensional Turing machine U with 2 states and 4 letters. Thus only the 2-state, 3-letter case for 1-head two-dimensional Turing machines remains open. Any positive or negative characterization of this case results in a complete characterization of the complexity of universal two-dimensional Turing machines. In addition, U leads at once to a universal 2-state, 2-letter, 2-head, 1 two-dimensional tape Turing machine.

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